DEPARTMENT OF MATHEMATICS
MAULANA AZAD COLLEGE

MathZ:N

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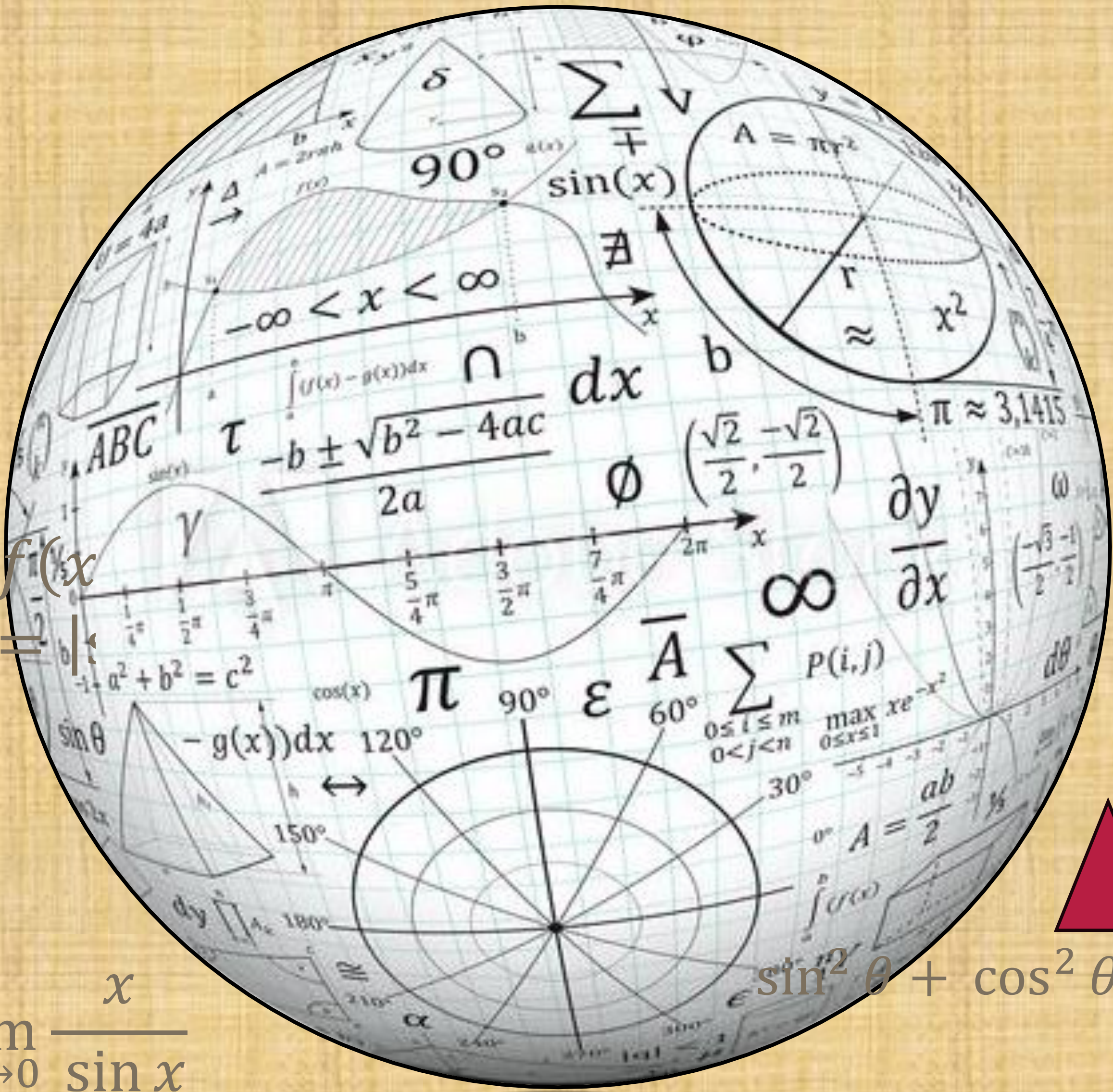


10 $\sin^2\theta + \cos^2\theta = 1$ $\lim_{x\to 0}\frac{x}{\sin x}$

2022-2023

$(a+b)^3$ $n!$

INDEX

PRINCIPAL'S MESSAGE

I am delighted to extend my warmest greetings to all the members of our esteemed institution as we present the latest edition of our Mathematics Magazine. It brings me immense pleasure to witness the dedication and enthusiasm that our students and faculty have put into curating this publication.

Mathematics is not just a subject; it is a language that unveils the secrets of the universe and empowers individuals with problem-solving skills that transcend the boundaries of academia. In this issue, you will find a diverse range of articles, problems, and insights that showcase the multifaceted nature of mathematics and its applications in various fields.

I would like to express my gratitude to the editorial team, comprised of both students and faculty members, for their tireless efforts in bringing this magazine to fruition. Their commitment to promoting mathematical knowledge and fostering a love for the subject among our community is truly commendable.

May this magazine serve as a source of inspiration, sparking curiosity and passion for the endless possibilities that lie within the realm of mathematics.

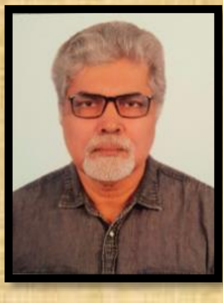
I extend my best wishes to the contributors and hope that this publication not only enhances our understanding of mathematics but also fosters a sense of camaraderie among us. Let us celebrate the achievements of our students and faculty, and continue to nurture a culture of learning and excellence in our institution.

Thank you for your continued support, and I look forward to witnessing the continued gro1wth and success of our mathematics community.

Principal
Maulana Azad College

Message from the
Head of the Department

Dr. Somnath Bandyopadhay,
Maulana Azad College, Kolkata



It is with great pleasure that I extend a warm welcome to each and every one of you to the vibrant realm of our College Mathematics Magazine. As the Head of the Mathematics Department, I am thrilled to witness the launch of this initiative that celebrates the beauty, diversity, and intellectual richness of the mathematical world.

Our College Mathematics Magazine is not just a publication; it is a testament to the collective passion, curiosity, and brilliance that defines our mathematics community. Mathematics is not merely a subject, but a language that unveils the secrets of the universe. Through this magazine, we aim to foster a deeper appreciation for the elegance and power that mathematics brings to our lives.

In the pages that follow, you will encounter a variety of articles, features, and insights that showcase the remarkable achievements of our students and faculty. This magazine serves as a platform to showcase the exceptional talent within our mathematics department.

I encourage each of you to actively engage with the content, whether you are a seasoned mathematician or just beginning your mathematical journey. This magazine is a space for everyone to learn, explore, and be inspired by the limitless possibilities that mathematics presents.

I would like to express my gratitude to the dedicated team of students and faculty members who have worked tirelessly to bring this magazine to fruition. Your commitment to excellence is evident in every page, and I am confident that this publication will be a source of pride for our mathematics community.

As we embark on this exciting journey together, let us celebrate the beauty of mathematics and the intellectual curiosity that drives our exploration of its depths. May this magazine be a source of inspiration, motivation, and a testament to the enduring spirit of our College Mathematics Department.

Wishing you all a fantastic experience as you delve into the pages of our inaugural Mathematics Magazine!

Head
Mathematics Department,
Maulana Azad College

TEACHER'S EDITORIAL

DR. NANDA DAS

DR. BABLI SAHA

Dear Readers,

It is with great pleasure and enthusiasm that we welcome you to the latest issue of Mathematics Magazine. As we embark on this mathematical journey together, I am reminded of the profound beauty and importance that mathematics holds in our lives.

In this edition, we have curated a collection of articles that span the diverse landscape of mathematics, from Algebra, Number Theory and Real Analysis in pure Mathematics to Mathematical Biology and articles related to Applied Physics. Our contributors, comprising of current and former students to the esteemed members of our mathematical faculty, have crafted insightful pieces that we are confident will engage, inspire, and perhaps even challenge your understanding of the mathematical universe. In addition to our regular features, this issue includes the various programs/extra-curricular activities that were held at the department.

Our goal is to not only showcase the breadth and depth of mathematical knowledge but also to demonstrate its profound impact on the world around us.

I extend my sincere gratitude to all the contributors who have shared their expertise and passion for mathematics, as well as to our dedicated editorial team for their tireless efforts in bringing this issue to fruition. I hope that Mathematics Magazine continues to be a source of intellectual stimulation and a catalyst for fostering a deeper appreciation of the mathematical sciences.

Nanda Das

Babli Saha

Associate Professor,
Mathematics Department,
Maulana Azad College

Associate Professor,
Mathematics Department,
Maulana Azad College

STUDENT'S EDITORIAL

-A Journey Beyond Numbers

Dear Readers,

Welcome to this edition of our college Mathematics Magazine! As we dive into the vast ocean of mathematical wonders, we find ourselves surrounded by the elegance, precision, and sheer beauty that mathematics offers. Mathematics is considered to be the most fundamental subject of all. Every other subject from Physics and Chemistry to social science like Economics to even the modern subjects like Computing and Data Science are all dependent on maths. In This issue, we embark on a journey beyond mere numbers, exploring the depths of mathematical concepts that not only shape our understanding of the world but also captivate our imagination.

This magazine has been the result of extensive hard work by the students of the Mathematics Department who were well guided and supervised in their writings and research about the various topics by our beloved Professors.

Through this magazine, our aim is to show that Mathematics is not just an abstract concept confined to textbooks and classrooms; it is a powerful tool with real-world applications.

We, the student editors of "MATHZIN" would like to express our sincere gratitude to all the students and Professors of our department for their valuable imputes. We hope the readers enjoy reading this edition of the magazine and find full of amazing facts. May the beauty of mathematics continue to inspire and captivate us all !

Best regards,

SRIJAN DAS

NABEELA JAHAN

ABHIJEET SAHA

STUDENT’S CORNER

A Brief Discussion bout π ~ Srijan Das

π, We all know this sign. It is Pi, a very common symbol, used by almost all branch of sciences. But what is it? We know it as a constant value, 3.14. But how we get this value? or is it true that π = 3.14? some of us say that it is equal to 22/7 , but is it true? Let us try to know about it…

The concept of pi is very old, difficulty of making a perfect cart wheel and calculating the area of a circle is the cause of generating pi. There is some trace of pi value in ancient India & in Egyptian civilization. At that time, they calculate the value of pi very nearly of it accurate value.

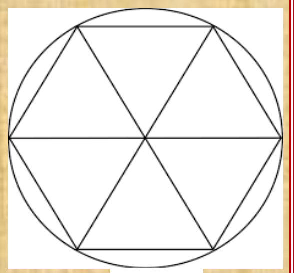


Figure 1

Around 250 BC Greek mathematician Archimedean improve a method to find the value of pi. He tried to calculate the perimeter of a hexagon (each side is 1 unit) inscribed in a circle [fig 1]. Then the perimeter of the hexagon is 6, which is less than the perimeter of the circle(say C) ,then 2 π r > 6, i.e, π > 3 [as r=1]. He also calculate the perimeter of the squre in which the circle [r=1] is inscribed [fig 2]. Then the perimeter of the circle is less then perimeter of squre.i.e, 2 π r < 8 => π <4.

i.e, 3 < π < 4

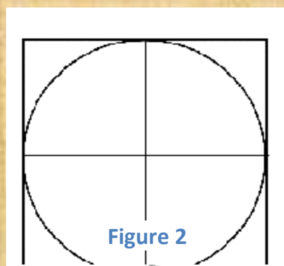


Figure 2

In this way he increase the sides of polygon , and see that the difference of the length of the perimetre of the circle andthe polygon is decreasing.

For 12 sided polygon 2 π > 6.212, and 2 π < 6.431 i.e, 3.106 < π < 3.2155.

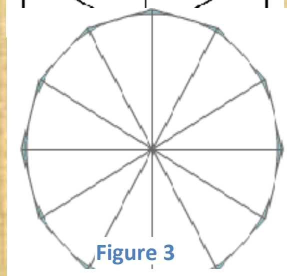


Figure 3

Increasing the sides he got at last 3.1408 < π < 3.1429.

After this many scientists try to increase the sides and calculate more accurately the value of pi. At 16^{th} century, Francois Viète compute with 393216-sided polygon then Ludolph van Ceulen work with 2^{62} sides and get 35 decimals of correct pi.

π =3.14159265358979323846264338327950288[it has been written on his grave]. Then 20 yrs. later Mr. Christof, who probably do this process last

and get 38 decimal accuracy.

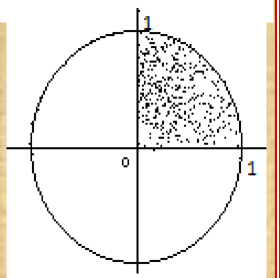


Figure 4

After this in year 1666 Newton (at 22 yrs. age when he was quarantining at home for Plague), he found that binomial series $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots$..is true for negative n also &$(1+x)^{-1} = 1 - x + x^2 - \cdots$,which is an infinite series, he also show that it works for fractions like n=1/2.

Now the equation of the upper half of a unit circle (area= π) is $y = (1-x^2)^{1/2} = 1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \cdots$

So, the area of the positive quadrant is $\frac{\pi}{4}$. Now sir Newton create the calculus and by integrating he try to calculate the area of the positive quadrant.

$$\frac{\pi}{4} = \left[x - \frac{1}{6}x^3 - \frac{1}{40}x^5 - \cdots\right]_0^1$$

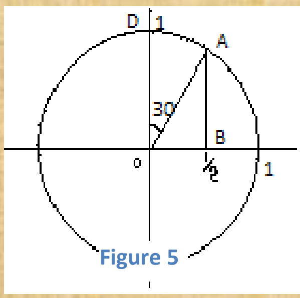


Figure 5

Putting 1 in x in this equation newton get a value of pi. But for x=1/ 2 the value of terms are getting smaller faster. So, he integrates the equation between 0 to ½. So the area of OBAD is $\frac{\pi}{12} + \frac{\sqrt{3}}{8}$ (<AOD = 30º, ΔAOB=$\frac{\sqrt{3}}{8}$).

$$\frac{\pi}{12} + \frac{\sqrt{3}}{8} = \left[x - \frac{1}{6}x^3 - \frac{1}{40}x^5 - \cdots\right]_0^{1/2}$$

Calculating this infinite series, the value of pi can be calculated easily. Calculating 1st five terms, we get Π = 3.1461and calculating only 50 terms we get Ludolph van Ceulen value which is accurate for 35 decimal place without calculate the perimeter of 2^62 sided polynomial. So, after this process no one try to calculate sides for getting pi value.

But now for calculating pi value there is many formulas one of them is Machin like formula which is

$$\frac{\pi}{4} = \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$$

using the series

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \quad , \qquad |x| < 1.$$

The latest record is 30 trillion digits on national PI-Day 14th March,2019 by using Chudnovseky algorithm which is

$$\frac{1}{\pi} = 12 \sum_{k=0}^{Inf} \frac{(-1)^k (6k)!\,(545140134k + 13591409)}{(3k)!\,(k!)^3 (640320)^{3k+3/2}}$$

.

Actually, pi is an irrational number so it cannot be reached to its accurate value by rational numbers. Basically, we have reached to a very accurate value of pi but this algorithm has been performed for checking super computers.

Now coming to the question that, is the value of pi = 22/7 ?

The answer is no. 22/7 is not equal to actual value of pi as 22/7=3.1428571429. That's why we use $4 * \tan^{-1} 1$ instead of using 22/7 in our programming code.

The Ramanujan Paradox - Somyajit Saha

$$1 + 2 + 3 + 4 + 5 + \cdots = -\frac{1}{12}$$

, Wait a minute, what? That looks very wrong. You must be wondering if the title is some horrendous typo, right? Well, let me assure you that it is absolutely not! The sum of all natural numbers is equal to -1/12. The equation above is actually a very important result used in theoretical physics, particularly in string theory. Now how can that be possible? Are physicists really that bad at mathematics? That can't be it! What is the proof behind this? Do we ever encounter it in real life? It is actually a rather simple proof. Before we get to that, it's important to understand a couple of other things.

Let's consider the following infinite summation:

X=1-1+1-1+1-1+…

Rearranging the above equation a little bit, we get:

X=1-(1-1+1-1+1-1+…)

If you look at the term inside the brackets, it in fact equals X. So, let's substitute that:

X=1-X, 2X=1, i.e. X=1/2

Now let's consider another sum.

Y=1-2+3-4+5-6+…

Writing it in another way, we get:

Y= 0+1-2+3-4+5-6+…

Adding the above two equations:

Y+Y= (1-2+3-4+5-6+…) + (0+1-2+3-4+5-6+…)

Grouping the corresponding terms in brackets,

We get:

2Y= 1+0-2+1+3-2-4+3+5-4-…

2Y= 1- (2-1) + (3-2) - (4-3) +…

2Y= 1-1+1-1+1-1+…

But the summation on the right-hand side is X.

Let's substitute it: 2Y=X, 2Y=1/2, Y=1/4.

Finally let's consider our original question at hand i.e. the sum of all natural numbers:

S=1+2+3+4…

We defined Y earlier as: Y=1-2+3-4+…

Subtracting Y from S we get

S-Y =1-1+2+2+3-3+4+4+…

i.e. S-Y= 4+8+12+16+…

we just calculated the value of Y to be ¼.

So, let's substitute it:

S-1/4 = 4 ×(1+2+3+4…) , S-1/4 =4S,3S=1/4,

S= -1/12

So, there we go! We now have the proof.

Why is it so counter intuitive?

The reason it looks counter intuitive is because people think “infinity” is actually a number, and that if we keep adding values until “infinity”, we will get a very large number. Well, this line of argument is wrong on multiple levels! First of all, “infinity” is not a number. It is a concept relating to uncountability. We have all these notions about numbers, absolutely none of which are applicable to “infinity”. The algebraic rules that apply to regular numbers do not apply to infinity. More specifically, the algebraic rules that apply to regular numbers do not apply to non-converging infinite sums.

Second of all you cannot keep adding values until infinity because you will never get there. So, if you try to prove it this way, you will actually never know what the sum of all natural numbers is. All of the sums we discussed above are non-converging infinite sums, so regular algebraic rules to not apply. It is like trying to use regular algebra to explain division by zero, which is going to get you nowhere. But mathematicians and physicists don’t like the concept of “getting nowhere”. So, there are ways to define the sums of non-converging infinite series so that they do not lead to contradictions. The one that leads legitimately to the conclusion that 1+2+3+4+5+…= -1/12 is called Ramanujan Summation.

Even if it’s not a big number, how on earth can it be negative fraction?

At first glance, it makes absolutely no sense. I agree to that! If you keep adding positive numbers to each other, the result has to be positive as well. This line of logic is completely valid, if we were not dealing with infinity! The concept of infinity is very fundamental, yet very obscure. When you think of a series of numbers and their summations, people are heavily inclined towards thinking about convergent series. When we are dealing with divergent series, things get a little trickier. For example, 1+ 1/2 +1/4 +1/8 +… is a convergent series because the sum approaches the value of 2 as you keep adding more terms. On the other hand, 1+1+1+1+1+… is a divergent series because the sum just keeps increasing as you keep adding more values to it.

What is the logical explanation?

For the more mathematically inclined readers, here is the explanation. To each convergent series, the operation that associates the limit of it’s partial sums is just a linear functional defined in the convergent series. This functional can be extended in many ways to the rest of all the series. This extension doesn’t have to have a meaning connected to that of the sum of convergent series. It is like when you have the function f(x)=1/(1-x) defined for x not equal to 1 (because 1/0 is not defined) and you extend it by defining f(1) =4 (or any other value). It is just an extension and it is not implying anything about 1/0. In real life, when you are designing functions, you have to accommodate these things so that your system behaves nicely.

This will be easier to explain with Riemann Zeta functions. This result is actually related to the Riemann Zeta function for s =-1. If you want, you can check out Wolfram Alpha for the Zeta function. If you set s=-1, the Zeta function will be reduced to the simple summation of 1+2+3+4+… and if you ask it compute the result, it will show it to be -1/12. You can read up more on Ramanujan Summation as well to have a better understand of the whole thing.

THE FASCINATING WORLD OF MAGIC SQUARE

- Chitrita Hazra

Magic squares have intrigued and captivated mathematician and enthusiasts for centuries. So, what is a magic square? A magic square grid filled with numbers, usually positive integers, arranged in such a way that the numbers in each row, column and diagonal add up to an equivalent sum. This sum is called "Magic Number". The 'order' of a magic square is the number of cells along one side, so a 3x3 magic square has 3 cells in each row and column. Let's look at some 3x3 magic squares.

2	9	4
7	5	3
6	1	8

Magic number=15

18	21	6
3	15	27
24	9	12

Magic number=45

Magic squares have a long history, dating back to at least 190 BCE in China. At various times they have acquired occult or mythical significance, and have appeared as symbols in works of art. In modern times they have been generalized a number of ways, including using extra or different constraints, multiplying instead of adding cells, using alternate shapes or more than two dimensions, and replacing numbers with shapes and addition with geometric operations.

- Now we try to unveil the mysteries of Magic Square step by step.

a	e	c
g	x	h
d	f	b

If we know the central number (i.e x) we can get the magic number 3x. If the magic number is 3x then from the table we can write a + b=2x, e + f=2x, c+d=2x, g+h=2x. Taking this concept we see an example

	3	
	5	
		2

Here x=5 so 3x=15, so from the previous table the remaining box of the second column will be 7(15-(3+5)=7).Putting this value, we get the blank box of the third row will be 4(15-(8+3)=6).By continuing this way, we get magic square below.

8	3	4
1	5	9
6	7	2

Let's see another where the central element is not given. Take a look at table below.

10		14
11		

Since the central element is missing here, let us assume it is x, then 10+14=2x, so x=12. Then 3x12=36 is the magic number. And by the previous method, the table becomes

15	8	13
10	12	14
11	16	9

❖ Some properties are given below

Magic constant

The constant that is the sum of any row or column or diagonal is called the Magic Constant or magic sum, M. Every normal magic square has a constant dependent on the order n, calculated by the formula $M=n(n^2+1)/2$. Since the sum of each row is M, the sum of n rows is the sum of 1,2,3,,………,$n^2$is $n^2(n^2+1)/2$. Since the sum of each row is M, the sum of n rows is $nM=n^2(n^2+1)/2$, which when divided by the order n yields the magic constant as $M=n(n^2+1)$. For normal magic squares of orders n= 3, 4, 5, 6, 7, and 8, the magic constants are respectively 15, 34, 65, 111, 175, and 260.

Magic square of order 1 is trivial

The 1x1 magic square with only one cell containing the number 1, is called trivial because it is typically not under consideration when discussing magic squares; but it is indeed a magic square by definition, if a single cell is regarded as a square of order one.

Magic square of order 2 cannot be constructed.

Normal magic squares of all sizes can be constructed except 2x2(that is, where order n=2).

❖ Some famous magic squares.

Luo Shu magic square

Legends dating from as early as 650 BCE tell the story of the Lo Shu (洛書) or "scroll of the river Lo".[8] According to the legend, there was at one time in ancient China a huge flood. While the great king Yu was trying to channel the water out to sea, a turtle emerged from it with a curious pattern on its shell: a 3×3 grid in which circular dots of numbers were arranged, such that the sum of the numbers in each row, column and diagonal was the same: 15. According to the legend, thereafter people were able to use this pattern in a certain way to control the river and protect themselves from floods[needs citation]. The Lo Shu Square, as the magic square on the turtle shell is called, is the unique normal magic square of order three in which 1 is at the bottom and 2 is in the upper right corner. Every normal magic square of order three is obtained from the Lo Shu by rotation or reflection.

Lo Shu from "The Astronomical Phenomena" (*Tien Yuan Fa Wei*). Compiled by Bao Yunlong in 13th century, published during the Ming dynasty, 1457–1463.

Magic square in Parshavnath Temple.

There is a well-known 12th-century 4×4 normal magic square inscribed on the wall of the Parshvanath temple in Khajuraho, India.[18][17][49]

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

This is known as the *Chautisa Yantra* (*Chautisa*, 34; *Yantra*, lit. "device"), since its magic sum is 34. It is one of the three 4×4 pandiagonal magic squares and is also an instance of the most-perfect magic square. The study of this square led to the appreciation of pandiagonal squares by European mathematicians in the late 19th century. Pandiagonal squares were referred to as Nasik squares or Jain squares in older English literature.


Magic Square at the Parshvanatha temple, in Khajuraho, India

Magic squares are fascinating mathematical constructs that have captured the imagination of people across cultures and centuries. Whether viewed as mystical symbols, mathematical curiosities, or artistic inspirations, magic squares continue to intrigue and inspire new generations to explore their secrets and beauty.

ক্যালকুলাসের দ্বন্দ্ব – রোহন মন্ডল

বিজ্ঞানের জগতে গবেষণার কৃতিত্ব নিয়ে প্রতিদ্বন্দ্বিতা নুতন নয়। তড়িৎ আর চুম্বকের পারস্পরিক লক্ষ করা নিয়ে জোসেফ হেনরি আর মাইকেল ফ্যারাডের বিবাদ, অক্সিজেন আবিষ্কারের কৃতিত্ব দাবি করে আতোঁ ল্যাভশিয়ে এবং জোসেফ প্রিস্টলির কাজিয়া, এডস রোগের জীবাণু শনাক্ত করা নিয়ে রবার্ট গ্যালো এবং লুক মতাঁনিয়ের দন্দ্ব সব এক একটা উপাখ্যান।

কিন্তু সব থেকে সেরা উপাখ্যান আজ থেকে কমবেশি তিনশো বছর আগের এক দ্বন্দ্ব। এর কারণ সেই দ্বন্দ্বের কেন্দ্রবিন্দু গণিতের এক অতি প্রয়োজনীয় শাখা, ক্যালকুলাস। আর প্রতিদ্বন্দ্বি দুজনের একজন হলেন পদার্থবিজ্ঞানের অবিস্মরণীয় বিজ্ঞানী আইজাক নিউটন। অপরজন হলেন গটফ্রিড উইলহেলম ফন লিবনিৎজ।

তবে গণিতের এই বিশেষ শাখার উদ্ভাবন নিয়ে নিউটন আর লিবনিৎজ কিভাবে দ্বন্দ্বে জড়ালেন তাঁর জবাব মিলবে ইতিহাসে।

১৬৬৫ সালের গোড়ায় হটাৎ প্লেগ এই মড়ক ছড়াতে থাকে ইংল্যান্ডে। কেমব্রিজ বিশ্ববিদ্যালয় বন্ধ হয়ে যেতেই নিউটন চলে আসেন তাঁর গ্রামের বাড়িতে। সেখানে কাটান প্রায় দু'বছর। এই সময়টা নিউটনের জীবনে এক আশ্চর্য কাল। পদার্থবিদ্যার যতগুলি আবিষ্কারের জন্য তিনি বিখ্যাত তার সব ওই সময় তার একক চিন্তার ফসল। এবং ওই সময়েই ক্যালকুলাস আবিষ্কার করেন তিনি। তবে আবিষ্কার করলেও নিউটন বিষয়টিকে ওই নামে চিহ্নিত করেননি। তিনি ডিফারেনশিয়াল আর ইন্টিগ্র্যাল ক্যালকুলাসকে বলেছিলেন 'ফ্লাক্সন' আর 'ফ্লুয়েন্ট' নির্ণয় প্রক্রিয়া।

তবে নিউটন তার অন্য সব আবিষ্কারের মতো এই আবিষ্কারও গোপনে রেখেছিলেন। আর ঝগড়ার সূত্রপাত হলো এখান থেকেই। ১৬৬৯ খ্রিস্টাব্দে নিউটন ছাত্র ও সহকর্মীদের জন্য লিখলেন এক পুস্তিকা। শিরোনাম 'দি অ্যানালাইজি পার একুয়েশনেস নুমেরো টারমিনোরাম ইনফিনিটাস'। এই পুস্তিকাতে নিউটন মাত্র একটুখানি ইঙ্গিত দিয়েছিলেন ক্যালকুলাস বিষয়ে তার কাজের।

এ দিকে ১৬৭৫ খ্রিস্টাব্দ নাগাদ লিবনিৎজ আবিষ্কার করলেন ক্যালকুলাস। সম্পূর্ণ নিজস্বভাবে। বলা বাহুল্য এক্ষেত্রে তিনি যে সব সাংকেতিক চিহ্ন ব্যবহার করলেন, যেমন dx, dy বা অন্যান্য আরো কিছু, সে সব বেশি জনপ্রিয় হল নিউটনের ব্যবহৃত চিহ্নগুলির চেয়ে।

১৬৮৪ খ্রিস্টাব্দের অক্টোবর মাসে লাইপৎজিগ বিশ্ববিদ্যালয় থেকে প্রকাশিত জার্নাল 'অ্যাকটা এরুডিটোরাম' এ এক প্রবন্ধ লিখে লিবনিৎজ প্রথম সকলকে জানালেন ক্যালকুলাস সংক্রান্ত তাঁর কাজ। এতে আতঙ্কিত হলেন নিউটন।

তাই ১৬৮৭ খ্রিস্টাব্দে প্রকাশিত তাঁর জগদ্বিখ্যাত গ্রন্থ 'ফিলোজফায়ে ন্যাচুরালিস প্রিনখিপিয়া ম্যাথমেটিকা'য় তিনি জানালেন নিউটন ক্যালকুলাসের লক্ষ্যে এগোচ্ছেন জেনে লিবনিৎজ ওল্ডেনবার্গ এবং কলিনস মারফত তাঁর কাছে অনুরোধ পাঠিয়েছিলেন তাঁর গবেষণার অগ্রগতির খবর লিবনিৎজকে জানাতে। ১৬৭৬ খ্রিস্টাব্দের অক্টোবরে নিউটন সেই চিঠির জবাবে ক্যালকুলাস বিষয়ে তাঁর গবেষণার বর্ণনা দেন ভাসাভাসা ভাবে।

এরপর ১৭ বছর দু পক্ষই নীরব থাকার পর ১৭০৪ খ্রিস্টাব্দে নিউটন প্রকাশ করেন আলোকবিদ্যা সম্পর্কে তাঁর গবেষণা সংবলিত বই 'অপটিকস'। এই বইয়ের পরিশিষ্টে কোনো প্রয়োজন ছাড়াই তিনি সবিস্তারে তাঁর উদ্ভাবিত ক্যালকুলাস। এবং সেইসঙ্গে সচেষ্ট হলেন এ গবেষণায় নিজের প্রথম সাফল্য প্রমানে।

'অপটিকস' এর পরিশিষ্ট পড়ে লিবনিৎজ খেপে গেলেন। ১৭০৫ খ্রিস্টাব্দে 'অ্যাকটা এরুডিটোরাম' জার্নালে বেনামে সমালোচনা লিখলেন বইখানার। এর প্রতিক্রিয়া সহজেই অনুমান করা যায়। ক্যালকুলাস যুদ্ধে ইউরোপের পন্ডিতগণ এবার যেন দ্বিধাবিভক্ত হল। নিউটনের পক্ষে ইংল্যান্ডের সমস্ত পন্ডিত। আর লিবনিৎজ এর সমর্থনে বাকি ইউরোপ। জার্মান পন্ডিতের পক্ষে এই বিপুল সমর্থনের অন্যতম কারণ নিউটনের নিজের গবেষণা গোপন করে রাখার জন্য ক্যালকুলাস এর আবিষ্কর্তা হিসেবে ইউরোপের অধিকাংশ দেশ ততদিনে কেবল লিবনিৎজকে চিহ্নিত করেছে।

এরপর ১৭১২ খ্রিস্টাব্দে, রয়্যাল সোসাইটির বিদেশী ফেলো হিসেবে, লিবনিৎজ সমিতিকে অনুরোধ করলেন আইডিয়া চুরির অপবাদ থেকে তাঁকে মুক্তি দেওয়া হোক। সোসাইটির প্রেসিডেন্ট তখন নিউটন স্বয়ং। পুরো ব্যাপারটা তদন্তের জন্য তিনি নিয়োগ করলেন এক কমিটি। তদন্তের পর কমিটি লিবনিৎজ কে নির্দোষ বা আইডিয়া তস্কর দুটোর কোনোটাই না বলে কেবল রায় দিল, নিউটন লিবনিৎজ এর ১৫ বছর আগে ক্যালকুলাস আবিষ্কার করেছেন।

তবে এই তদন্তের রিপোর্টেই ক্ষান্ত হল না ক্যালকুলাস যুদ্ধ। চলল মোট প্রায় চার দশক। এমনকি দুজনের মৃত্যুর পরেও। জীবদ্দশায় বিজ্ঞানী হিসেবে খ্যাতির শীর্ষে পৌঁছলেন নিউটন। মৃত্যুর পর লন্ডনের ওয়েস্টমিনিস্টার অ্যাবে তে পূর্ন রাষ্ট্রীয় মর্যাদায় সমাধি দেওয়া হল তাঁর মরদেহ।

আর লিবনিৎজ গণিত গবেষণায় আর তেমন ব্যস্ত রইলো না। বই লিখলেন দর্শনশাস্ত্রের কিছু কিছু বিষয়ে। মাত্র সত্তর বছর বয়সে তাঁর মৃত্যুর পর মরদেহ সমাধিস্থ করার সময় হাজির ছিলেন কেবল তাঁর সেক্রেটারি।

তথ্যঋণ : (১) ইন্টারনেট।

(২) পথিক গুহর লেখা,'ঈশ্বরকণা মানুষ

ইত্যাদি'।

ALUMINI’S CORNER

- Flint Hills Series — Arka Prabha Roy
- Extended Complex Plane and Compactness. — Arka Prabha Roy
- Let’s See the Ecology through Physics — Ayan Paul
- How Many Colours Do We Need? A Survey of Four Colour Theorem and Its Generalizations. — Arghya Singha
- **Beauties of Arithmetic Regression Topology On *Z*** — Dipanjana maity
- An Introduction to Cantor Set. — Bishyay Majumder

FLINT HILLS SERIES

Arka Prabha Roy

Anyone in the field of mathematics, from students to researchers to teachers, has always come across terminologies such as "Series of Real Numbers" and "Convergence" in undergraduate level, and analysed the series to conclude if the series is convergent or divergent. They have learned a set of methods such as "Root test", "Ratio Test", "Condensation test" and so on. But as Sir Isaac Newton once said What we know is a drop, what we don't know is an ocean. And thus there exists a few series which might pique the reader's interest, one such being the Flint Hills Series.

Named after a region in Eastern Kansas, this infinite series of real numbers is expressed as

$$\Sigma \frac{1}{n^3 sin^2 n}$$

Now, what is so interesting about this series? It is the fact that the convergence of this series is unresolved, meaning we don't know if it will converge or diverge. But why? One might think initially that in the denominator, the n^3 will dominate the $sin^2 n$ part and this will converge. But the flaw of this approach is $f(x) = \frac{1}{sin^2 x}$is not bounded. Now one can argue that for no natural number n, $sin^2 n$ is zero and thus $\frac{1}{sin^2 n}$is a finite number , and although the statement is true, the argument isn't valid in this case because for certain $n \in N$, $\frac{1}{sin^2 n}$takes large value. So can you say that this is a divergent series? The answer is no, because it is unknown if the cardinality of the collection of all those certain n is finite or not. And for the rest natural numbers, $\frac{1}{n^3}$dominates$\frac{1}{sin^2 n}$.

One may also try to do a few tests, which will fail to give us any conclusions to its convergence and divergence. One can also take a graphical approach to see the behaviour of this series, but it will not help them as no new information can be obtained which isn't stated above.

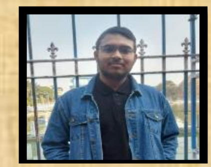
But now the question pops up, why? Why is the series showing erratic behaviour for certain natural numbers? The key lies in one of our well-known irrationals (and transcendental) real numbers, which is Pi (π).We know, $sin^2(m\pi) = 0$ for all integers m. So, if you have an integer which is close to mπ, it will produce a large value of $\frac{1}{sin^2 n}$. An example being the elements of sequence 1,3,22,333,355…. and so on. This sequence is indexed A046947 in OEIS, which generates a subsequence of $|sin(n)|$ which monotonically decreases to 0. Along with the n^3 factor multiplied in the denominator; the series shows an erratic behaviour due to which convergence of the series is neither proved nor disproved.

Although not proven, a statement is presented by Dr. Max Alekseyev in 2011. He commented that if Flint Hills Series converges, that would imply irrationality measure of π, denoted by $\mu(\pi)$satisfies $\mu(\pi) \leq 2.5$, which is a much stronger condition than the best known upper bound $\mu(\pi) \leq 7.6063\ldots$..

And there you have it, a series of real numbers whose convergence is unknown. The reader can take this series up and try to see it for themselves. Who knows, you may be the one to solve this "Unsolved" problem.

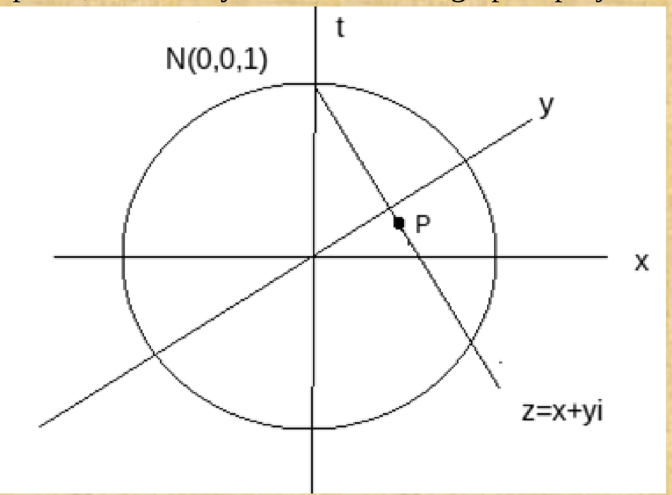
Extended Complex Plane and Compactness:

Arka Prabha Roy



You've heard of complex plane, right? If not, think of it as the real plane and denote $1 = (1,0)$ and $i = (0,1)$, where i is the square root of -1. And you can see that this plane has some special properties regarding its subsets and functions on it. But we're interested in talking about the Extended Complex Plane. Now you may ask "what do I mean by Extended". When we talk about Complex plane, we do not include infinity in it. But in Extended complex plane, we extend the complex plane by adding ∞ with the set. This set is denoted by $\mathbb{C}\cup\{\infty\}$.

But what exactly does it look like? You can't visualise infinity, at least not in the same dimension. One can visualise infinity from a higher dimension. That's where the Riemann Sphere comes in play. It's pretty simple, you first draw the x-y plane (complex plane). Then from origin, perpendicular to the plane, z axis can be drawn, which will be our third dimension. Now take a unit sphere centred at origin, and think of its highest point as North Pole, or N(0,0,1). Now for a complex number z=x+iy, or (x,y,0), draw a line from the North Pole to the z. The point P at which the line intersects the sphere (except N) is the projection of the complex number on the Riemann sphere, commonly known as stereographic projection.



Now, it is evident that no complex number maps to the North Pole. But assume if infinity comes into play now. Then a line from infinity will be parallel to the complex plane, only intersecting the sphere at North Pole. Then we can say that infinity maps to N and we will get a bijection from $\mathbb{C}\cup\{\infty\}$ to the unit sphere. This is also bicontinuous, so we have a homeomorphism here. Now the topological properties of $\mathbb{C}\cup\{\infty\}$ will be the same of unit sphere. One of these properties is Compactness. We can say that $\mathbb{C}\cup\{\infty\}$ is compact, and this is an example of one point compactification. But let's prove it by the definition involving covers.

Suppose A be an open set containing ∞. Then the set A has to be of the form
A= {z: |z|>M where M is a fixed real number}. Then suppose $\{U_n\}$ be a collection of open covers covering the Extended complex plane. Then one of them has to be of the form A. Then we can take finitely many open covers of the collection, which will cover the closed set $\{z: |z| \leq M\}$ and add the single set of form A to the finite collection, which will give us a finite subcover of the initial cover. Then we can conclude that $\mathbb{C}\cup\{\infty\}$ is compact.

This is one of many properties of the extended complex plane. Now you can also take this set up and check its various properties yourself.

LET'S SEE THE ECOLOGY THROUGH PHYSICS

~Ayan Paul

The story I will narrate here is the manifestation between the population ecology and

classical mechanics. Whenever we hear the term "Ecology" or "Population Ecology", the most common things that appear in our mind are *Ecosystem, Growth curves, Prey-predator interaction, Symbiosis, Parasites, Plankton experiments,* and many more. Since the subject “Ecology” is primarily comprised of several biological and natural phenomena. But, all of a sudden, we fairly miss an important aspect behind the aforementioned incidents, i.e., the inherent mechanism of the ecosystem. We all know that the dynamics of a natural system is regulated by various physical laws, which are not exceptional in case of the ecological instances. Let me give you a brief example of this.

Suppose we consider the paradigm of the famous Malthus law in ecology. The mathematical expressions of the Malthusian dynamics show that nature will allow all the species for unlimited growth. That means if nature does not provide any restrictions, the species can grow unboundedly. Doesn't the statement of Malthus law sound like Newton's first law in classical mechanics? Yes, of course.

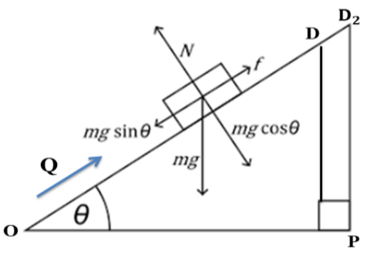


Fig 1: Pulling of an object through an inclined plane. Here Q, mg, θ, f, N denote the external force, weight of the object, inclination angle, frictional force, and normal component of the plane respectively.

But, you will definitely wonder to think how I can suddenly jump into Newton's law while discussing an ecological phenomenon. Since, the laws of Newton are completely defined on the particle motion. Actually, if anyone recalls the first law of Newton, i.e., "if a body is at rest or moving at a constant speed in a straight line, it will remain at rest or keep moving in a straight line at constant speed unless a force acts upon it.", he/she definitely finds a synergy between the particle dynamics and the Malthusian effect. Since, the statement of Malthusian law highlights that, if nature will not provide any kinds of perturbations to a species, it will grow unboundedly in an exponential fashion. But, in reality, this phenomenon does not happen since several external affairs exist in an ecosystem, viz. competition, cooperation, mutualism, etc., just like the forces on any particle body. Instead of several external involvements, any species must follow a sigmoidal or S-shaped growth trajectory to reach its destination, i.e., carrying capacity or asymptotic size. The reason to attain the carrying capacity is very phenomenological since every species' intrinsic nature to sustain around its asymptotic size.

It is worth mentioning that the S-shaped growth profile can be mostly classified into three major steps. On the initiation of the growth process, it can be distinguished into *Lag, Log,* and, *Stationary phases*. That means at the end of every stage, the species must get a kicked-up force in terms of its internal metabolism or population structure to attain the next phase of the growth process. It seems to be a particle crossing one to another inclined plane. So, you can think that biomass changing over a growth trajectory can be synergistic with pulling an object in an inclined plane by providing some external force parallel to the axis of that plane. So, it is better to discuss this synergistic phenomenon through an example of pulling an object through the inclined plane.

Let us consider an inclined plane ODP with an inclination angle θ(the figure is given below). Now, the aim is to lift an object (M) from the position 'O' to 'D'. To do this, you must provide an external force 'Q', acting parallel to the plane 'OD'.

Hence, you can think that the smooth sigmoidal trajectory of the species growth can be approximated by the cumulative effect of three distinct inclined planes ABJ, BCR, CES in the figure 2. Since the sigmoidal trajectory contains three growth phases: lag, log, and stationary. But, in this case, three planes must have a different inclination angles. But, one can generalize the concept of three inclined planes into a multiple for a better approximation of the sigmoidal trajectory. When any individual is lifting the object 'M' by the external force 'Q', the frictional force and the component of the weight of the object are acting on the object in the opposite direction.

Similarly, when species biomass changes over time in a sigmoidal trajectory through the growth process, the species need food support; this is equivalent to the concept of the external force in particle dynamics of the inclined plane. For changing the biomass in the sigmoidal trajectory, the species need sufficient food support and have to overcome the issue of competition. So, competition is the only negative force acting on the species against its natural growth process. This competition term is equivalent to the frictional force and the weight component of the object of the particle dynamics in an inclined plane. It needs a minimum external force to lift the object 'M' from the position 'O' to 'D'. If you apply less force, the particle will return to its original position due to its component. But, if you reduce the inclination anglesay, α ($< \theta$), the same force can lift the object to a positionD_1, below the point'D' (see the figure 3). This phenomenon is pretty similar to the species growth process. For a species, we generally have a reasonable size at maturity, which is favorable for its successful reproduction. The species need a favorable amount of food concentration to achieve this size. Below this food concentration, the species maturity size is reduced, which is not in favor of its reproductive process. This stage is comparable with the phenomenon when the object 'M' has reached the point 'D_1' instead of the 'D' due to the reduced external force. If you increase the food concentration, the size at maturity does not change substantially. This is also similar if you increase the external force instead of the minimum force required to lift the object in the inclined plane. The object will reach the same point, 'D', but the passage time is reduced.

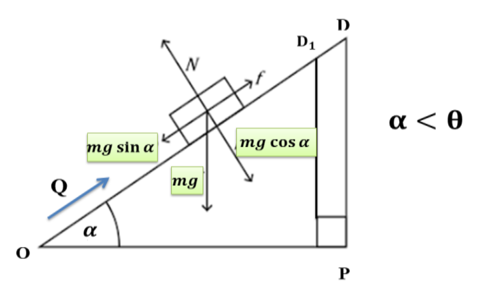


Fig 3: Pulling of an object through an inclined plane. Here Q, mg, α, f, N denote the external force, weight of the object, inclination angle, frictional force, and normal component of the plane respectively. Note that the angle θ is connected from the Fig 1.

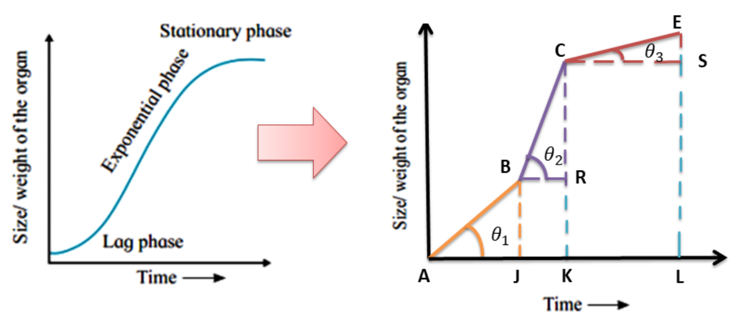


Fig 2: The approximation of the sigmoidal growth curve to three cumulative inclined planes

In classical mechanics, if you increase the amount of the external force on the object 'M', the inclination angle also increases. Then, the object will reach a new point, 'D_2', located above the point 'D' (see the figure 1).

Compared with the species growth process, as we already know, every species has a limit in reaching its maximum size at maturity. Hence, reaching the species maturity size will no longer be affected despite providing more and more food concentration beyond the most favorable, responsible for attaining the maximum size at maturity. Additionally, this also happens due to variations in the species metabolic activities and behavioral discrepancies among different individuals of the same species. In another way, variations in metabolic activities depend upon the differences in the food concentration, which directly impacts the species growth. The system with low food concentration and a fixed number of predators faces more intra-specific competition than the system with more food concentration and the same number of predators. These different levels of intra-specific competitors trigger differences in the species metabolic activities under different food concentrations. However, in the case of the particle dynamics, as there are no issues of the interference of several biological factors like intra-specific completion, variations in metabolic activities, and the limit in achieving the maximum size at maturity of the predator species, etc., the object in an inclined plane can reach at the point 'D_2', when the provided external force is more than the previous one.

How Many Colours Do We Need? A Survey of Four Colour Theorem and Its Generalizations

Arghya Sinha

1 Introduction

When we colour the countries on a world map, we usually want to use different colours for adjacent countries to distinguish the measily. For example, India and Nepal should have different colours on the map. The Four Colour Problem asks how many colours are enough to colour any map in this way. It turns out that the answer is four and this is known as the Four Colour Theorem. In this article, we will explore this fascinating result, its mathematical foundations, and some extensions and generalizations to other types of maps and surfaces.

2 Four Colour Problem

2.1 Statement

Before going into the statement of four colour problem, we go through some essential preliminary definitions to state the problem.

Let S be a Surface and A_i , $i \in \Lambda$ (index set) be open, connected subsets of S such that

- For any i, j ($i, j \in \Lambda$), $A_i \cap A_j = \phi$
- For all $i \in \Lambda$ boundary of A_i is a simple closed curve.

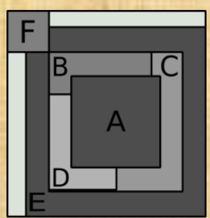
Definition:1(Country). For each $i \in \Lambda$, A_i is called a Country of S.

Definition:2(Adjacent Country). We call two countries adjacent if they share a common boundary segment.

Definition:3(Map). For any $i \in \Gamma \subset \Lambda$, Γ^{A_i} is called a map of S.

InFigure1, A, B, C, D, E, and Fare all countries in the plane. A is an adjacent country to B, C, and D. F is adjacent to E. Note that F and B are not adjacent countries as they do not share a boundary segment. We can consider the union of A, B, C, D, E, and F as a map in the plane. Now, we are ready to state the Four Colour Problem. The statement is as follows.

The countries of any given map of a sphere can be coloured using only four colours in such a way that no two adjacent countries are coloured with the same colour.



Remark. The first attempt stop rove this conjecture we remade by P. G. Taitin1879 and A. B. Kempe, an English barrister, in 1880. However, their proof were flawed, as P. J. Heawood, an English mathematician, demonstrated in1890 by finding a gapin Kempe'sargument. The conjecture eremainedun-

Proven until 1997 when Roberts on, Sanders, Seymour, and Thomas published a proof that relied on extensive computer calculations. A more elegant proof was given by Georges Gonthier [1] in 2005, using general-purpose theorem-proving software.

Now, we will look into a simpler approach to this problem using Graph Theory. Figure1: Simple r statement of Four Colour Problem

By using Graph Theory, we can see this problem in a new way, which will eventually help us to solve a more generalized version of the problem called Map Colour Theorem. Before that, we need to recall the definition of the planner graph.

Definition 4 (Planer Graph). A planer graph is a graph that can be drawn on a plane in such a way that its edges only intersect at the vertices.

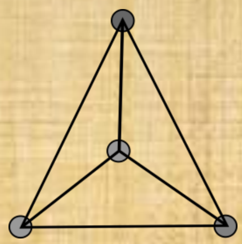


Figure 2: Example of a Planer Graph

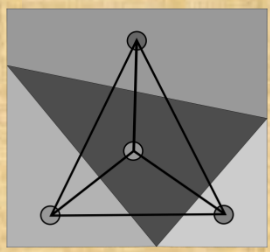


Figure3: Map corresponding to a Planer Graph

Now, we can visualize any map as an un-directed graph such that for each country, a unique vertex of the graph lies on the country. This graph is a *Planer Graph.*

Conversely any planer graph can be visualized as a map similarly.

So, a map always exists corresponding to a given planer graph, and the correspondence is not one-to-one but onto. We say *A planer graph is n-colourable* if all the corresponding maps the planer graph can be coloured with n colours in such a way that no two adjacent countries have the same colour. In Figure 3, we have drawn the maps we get corresponding to the planer graph of the example of Figure 2. Also, we coloured the map using four colours.

Thus, we can state the four colour problem as *Any Planer Graph is four-colourable.*

Now, we try to generalize the concepts of the four colour problem (Which is true for maps on Sphere) to other surfaces.

3 Map Colour Theorem

3.1 Chromatic Number

A *surface* is a two-dimensional object that can be embedded in a three-dimensional space. Some examples of surfaces are spheres, tori, and cylinders. A surface is *closed* if it has no boundary and is compact, meaning of interregional of space can contain it. One way to construct closed surfaces is by attaching handles to a sphere. A *handle* is a tube-like shape connecting two sphere points. A genus-*n* surface is a closed surface with *n* handles attached. The genus of a surface is a measure of its complexity or number of holes. A sphere has no handles or holes, so it is a genus-0 surface. A torus has one handle and hole, making it agenus-1 surface. We can denote a genus-*n* surface by A_n , where *n* is a non-negative integer.

Theorem1 (Classification Theorem for Orientable Surfaces). *Any closed orientable surface is homeomorphic to* S_n *for some non-negative integer n.*

Definition:5 (Chromatic Number). A chromatic number of a surface *S* is the number *n* such that any map on *S* can be coloured with *n* number of colours, but there exists a map *M* on *S* such that n_1 colours cannot colour *M*. The Chromatic Number of a surface *S* is denoted by χ(S)

The Four Colour Problem precisely states that *The Chromatic Number of the Sphere is four, i.e.* $\chi(S_0)$ =4.

3.2 Map Colour Theorem

Since from Classification Theorem we know that any closed orientable surface is homeomorphic to some genus-*p* surface, It is sufficient to look into S_p, $p \in \mathbb{Z}^+$only.

3.3 Similar Statement for Non-Orientable Surfaces

We take a rectangle of the form illustrated in Figure 4. Draw arrows similarly and mark 1,2,3,...,*q*,*q*,*q*1,̄...,1.Nowattach1with 1, 2 with 2, ...*q* with *q* in such a way that after attaching, the arrow shave the same direction.

This way, we will obtain a surface called

Mobius Strip of order q
And denote it by M_q.

In the figure above, we would obtain M_3, and clearly, the Mobius Stripwe are familiar with is M_1.

Figure4: Mobius Strip of Order 3

We can calculate the Chromatic Number of M_q using the follow-ing formula proved by G.Ringel [3]in1954.

$$\chi(M) = 7 + \sqrt{1 + 24\,p}, for\ q = 2 \qquad (3)$$

$$\chi(M_2) = 6 \qquad (4)$$

4. Conclusion

he Four Colour Problem is a classic example of how a seemingly simple question can lead to profound discoveries in mathematics. In this article, we have explored some of the history, methods, and extensions of this problem. To understand the proofs of theorems mentioned in the article, one needs to understand the topological properties of surfaces and graphs. An interesting concept in this regard is embedding a graph on a surface where the graphs can be seen as a one-dimensional skeleton of a surface. These results demonstrate the richness and diversity of the mathematical field that studies the interactions of shapes, spaces, and colours. We hope this article has sparked the reader’s interest and curiosity in this fascinating research domain.

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BEAUTIES OF ARITHMETIC ROGRESSION TOPOLOGY ON Z

~Dipanjana Maity

1. Introduction

Topology is a structure of a set, which is invariant under continuous deformation that can be restored. This deformations are Homeomorphisms and Homotopies. On a same set, we can define various topologies such that the properties and behaviors of the topological spaces are fully different. Here we will study a very interesting topology on the set of integers Z, which is constructed by taking all Arithmetic Progressions. We will observe that how we can prove the results of this *Arithmetic Progression Topological Space* by using the concepts of Cosets in Group Theory.

Topology and Number Theory are two different branches of Mathematics. But is there any relation between them? We know that, in 300 BC, ancient Greek mathematician Euclid proved the Fundamental statement in Number Theory, which tells us that there are infinitely many primes in N. Can we prove the infinitude of primes in N, by using Topology? Surprisingly, the answer is yes. In my short write up, I will try to explain this topological proof of *Infinitude of Primes in N*.

2. Preliminaries

Before moving on to the main part, let's remind some basic topological concepts.

- Topological Space : Let X be a non empty set. A topology τ on X is a family of subsets of X such that (a) τ contains X and ϕ,
  (b) τ is closed under arbitrary union and finite intersection.

  Elements of τ are called open sets. Compliment of a open set is called closed set.
- Basis: Let X be a non empty set. A basis for a topology on X is a family B of subsets of X such that
  (a) For each element x ∈ X, there is an element B ∈ B such that x ∈ B.
  (b) If x ∈ $B_1 \cap B_2$ for some $B_1, B_2 \in B, then\ there\ exists\ B_3 \in B\ containing\ x\ such\ that$ $B_3 \in B_1 \cap B_2$.

By taking arbitrary union of elements of a basis, we get the open sets i.e. elements of the topology.

- Hausdorffness : A topological space (X, τ) is called Hausdorff or T_2, if any two distinct points of X can be seperated by disjoint open sets of (X, τ), $i.e.$, there is two disjoint open sets in (X, τ), containing these two points respectively.
- Regularity: A topological space (X, τ) is called Regular, if a closed set and an arbitrary point, not in the set, can be separated by disjoint open sets of (X, τ).
- Normality: A topological space (X, τ) is called Normal, if any two disjoint closed sets can be separated by disjoint open sets of (X, τ).
- Second countability: (X, τ) is called Second countable, if it has countable basis.
- Metrizability: (X, τ) is called Metrizable, if we find a metric d on X such that open balls of (X,d) are basic open sets of (X, τ) and vice versa.
- Connectedness: (X, τ) is called Disconnected, if it has a non empty, proper clopen set. Otherwise, (X, τ) is called Connected. (X, τ) is Totally disconnected, if singletons are only connected sets.

- Compactness: (X, τ) is called Compact, if every basic open cover of (X, τ) has a finite subcover.

3. Arithmetic Progression Topology

Let us discuss about the the Arithmetic Progression Topology -

- Furstenberg's Topology on Z :

In 1955,Harry Furstenberg introduced the Arithmetic Progression Topology on the set of all integers Z , taking all possible Arithmetic Progressions on Z as basis.

- Basis:- Let $B = \{A_{m,n} : m \neq 0, m, n \in Z\}$, where $A_{m,n} = \{mk + n : k \in Z\}$.

Then B is a collection of subsets of Z.
Now, observe that, the set $A_{m,n} = \{mk + n : k \in Z\} = mZ + n$
Then $B = \{mZ + n : m \neq 0, m, n \in Z\}$,
i.e., B is nothing but the collection of Cosets of all normal subgroups nZ of Z .
Let's try to prove that B is a basis for some topology on Z.

Firstly, $A_{1,0} = Z \in B$.
Secondly, let $x \in A_{m_1,n_1} \cap A_{m_2,n_2}$.
We claim that, $A_{m_1,x} \subseteq A_{m_1,n_1}$. In fact, $m_1k_1 + x = m_1(k_1 + k_2) + n_1 \in A_{m_1,n_1}$,for any integer k_1 and for some integer k_2.
So, our claim is true. Similarly, $A_{m_2,x} \subseteq A_{m_2,n_2}$.
Now, $m = lcm\{m_1, m_2\}$. Then, we claim that, $A_{m,x} \subseteq A_{m_1,x} \cap A_{m_2,x}$.
In fact, $mk + x = p_1m_1k + x = p_2m_2k + x \in A_{m_1,x} \cap A_{m_2,x}$, for any $k \in Z$ and for some $p_1, p_2 \in Z$. so, our claim is verified. Hence, $x \in A_{m,x} \subseteq A_{m_1,x} \cap A_{m_2,x} \subseteq A_{m_1,n_1} \cap A_{m_2,n_2}$.

So, <u>B is a basis for some topology on Z.</u> Now, taking all possible arbitrary union of $A_{m,n}$ $(m \neq 0)$,
we get the open sets of a topology τ_F on Z ,namely, <u>Furstenberg's Topology on Z</u>.

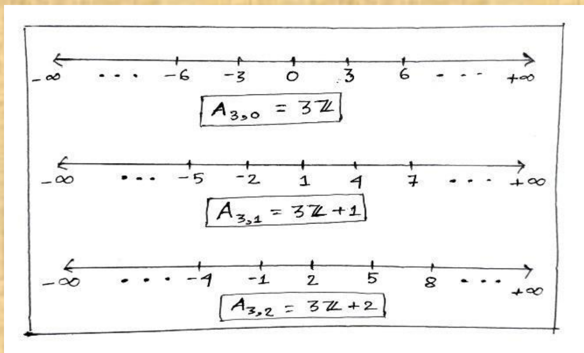
- <u>Basic open sets are closed:-</u> A very interesting point is that, every basic open set $A_{m,n}$ is closed here.

Let $A_{m,n} = mZ + n$ be an arbitrary basic open set.
Then, $n \equiv r(mod\ m)$ for some $r \in \{0,1,2, \dots, m-1\}$.
Then by using concepts of Coset, it can be says that, $A_{m,n} = mZ + n = mZ + r = A_{m,r}$.
Again, since $mZ + r$ is nothing but a coset of mZ,

$$\cup_{i=0}^{m-1} (mZ + i) = Z \Rightarrow mZ + r = Z \setminus (\cup_{i=0, i\neq r}^{m-1} (mZ + i)).$$

So, $A_{m,n} = mZ + r$ being compliment of a open set is closed.

For example, $3Z \cup (3Z + 1) \cup (3Z + 2) = Z$, so, $3Z, 3Z + 1, 3Z + 2$ are both open and closed.
From the picture given below, the concept will be easy to visualize:



- (Z, T_F) is Hausdorff:- The Furstenberg Topological space is Hausdorff of T_2.

> Let $n_1 \neq n_2$ be two integers.Take a integer $m > max\{|n_1|, |n_2|\}$.
> Then $A_{m,n_1} = mZ + n_1$ and $A_{m,n_2} = mZ + n_2$ must be disjoint, since they are two distinct cosets of mZ. And clearly $n_1 \in A_{m,n_1}$and $n_2 \in A_{m,n_2}$.

- (Z, T_F)*is* totally disconnected:- Since every basic open set is closed in (Z, τ_F) , this topological space is disconnected. Even it can be easily proved that the Furstenberg Topological space is Totally Disconnected.

> Let S be a subset of Z which contains at least two distinct points n_1, n_2.
> Take a integer $m > max\{|n_1|, |n_2|\}$. Then $\cup_{i=0}^{m-1} (mZ + i) = Z$ and $|n_1|, |n_2| \in \{0,1,\ldots, m-1\}$.
> So, $S = ((mZ + |n_1|) \cap S) \cup ([Z \backslash (mZ + |n_1|)] \cap S)$ is a disconnection of S.

- (Z, T_F) is Metrizable:- Being countable basis, (Z, τ_F) is second countable. We already saw that (Z, τ_F) is Hausdorff.

  Again, let x $\in Z$ such that $x \notin E$, where E is a closed set in (Z, τ_F). Then, $Z \backslash E$ ia a open set containing x. So, there exists a basic open set $A_{m,n}$ (containing x), contained in $Z \backslash E$. Now, $A_{m,n}$ *is closed also*. So, we get two disjoint open set $A_{m,n}$ *and* $Z \backslash A_{m,n}$ containing x and E respectively. Hence (Z, τ_F) is Regular.

  Then by Uryshon's metrization theorem, (Z, τ_F) *is Metrizable*.

- (Z, T_F) is non compact:- If we consider the basic open cover $\{2Z + 1, 3Z, 4Z + 2, 8Z + 4, 16Z + 8, \ldots\ldots.\}$ of Z, but it has no finite subcover which contains every power of 2. So, the space is non compact.

4. Proof of Infinitude of Primes by using Furstenberg Topology

By this time, we have studied some of the behavior of Furstenberg Topological space. It is now time to observe the most interesting fact that, Furstenberg topological space can easily prove the infinitude of primes.

❖ There are infinitely many primes in N:

Let P be the set of all primes in N. Now, just notice two important facts -

(i) Every non empty open set of the Furstenberg topological space is infinite.

(ii) $\cup_{p \in P} pZ = Z \setminus \{-1,1\}$

Now, if possible, P is finite set.

since finite union of closed sets is closed , $\cup_{p \in P} pZ$ is closed. That means, {-1,1} is open,

which contradicts that every open set in (Z, τ_F) is infinite.

Hence, P is an infinite set, i.e., there are infinitely many primes in N.

5. Conclusion

There are strange mystery hidden in every branch of Mathematics. Sometimes They appear in front of our eyes to surprised us so much. In this write up, i tried to discuss such a amazing collaboration of Topology with Number theory, given by Furstenberg. Two another Arithmetic Progression Topologies are Golomb topology and Kirch topology on N respectively.

The basis of Golomb Topology or relatively prime topology on N is $\{mN_0 + n : m, n \in N, gcd(m,n) = 1\}$,here $N_0 = N \cup \{0\}$.The subbasis of kirch Topology or prime integer topology on N is $\{pN_0 + n : p \text{ is prime and not divide } n, n \in N\}$. This two topologies also gives us so many interesting facts. For detailed discussion, one can see the references :

1. *A connected Topology for the Integers by Solomon W. Golomb*
2. *A countable, connected, Locally connected Hausdorff space by A.M. Kirch*

AN INTRODUCTION TO CANTOR SET

~ Bishyay Majumdar

INTRODUCTION

The beauty of mathematics lies in its nature to surprise us i.e.; very often a phenomenon occurs in some area of mathematics where one would expect it least to occur. And in this process, seemingly so distinct two areas of mathematics get connected by some bridge. The Cantor set, named after George Cantor, is one of those unexpected examples that has amazed mathematicians for over a century. Let us first think about two questions: (a) Can we remove finitely many disjoint sub interval from an interval, say [0,1], such that we still be left with an uncountable set? And secondly, (b) Can we remove infinitely many disjoint sub interval from [0,1] to have the same result? Well, the answer to both the questions is yes. The first one is more obvious than the second. To answer the second one, one can take the intervals of the form $I_n = \left[\frac{1}{2n}, \frac{1}{2n-1}\right]$, and then take the set $[0,1] \setminus \cup_1^\infty I_n$ to get the desired result. Now, can we discard these sub-intervals in a way such that the set we are left with is uncountable but has zero measure? We will discuss what a measure of a set is later, but for now just think that a set is of measure zero implies the set is present almost nowhere. Can such a set contain not only infinite but uncountably many elements (i.e.; same number of elements as R)? Surprisingly, the answer is yes. The construction of the Cantor set provides us with the proof. Apart from set theory and Measure theory Cantor set also has interestingly occurred in topology and in some other areas of mathematics. In this write up, our main objective is to study some basic properties of the Cantor set.

1. CONSTRUCTION

Henry John Stephen Smith first discovered the Cantor set in 1875 and George Cantor described it in 1883. Let us give the mathematical construction to Cantor set as following manner:

Let, $E_1 := \left[0, \frac{1}{3}\right] \cup \left[\frac{2}{3}, 1\right]$ (i.e.; we are removing the interval $\left(\frac{1}{3}, \frac{2}{3}\right)$from [0,1].)

$E_2 := \left[0, \frac{1}{3^2}\right] \cup \left[\frac{2}{3^2}, \frac{1}{3}\right] \cup \left[\frac{2}{3}, \frac{7}{3^2}\right] \cup \left[\frac{8}{3^2}, 1\right]$.

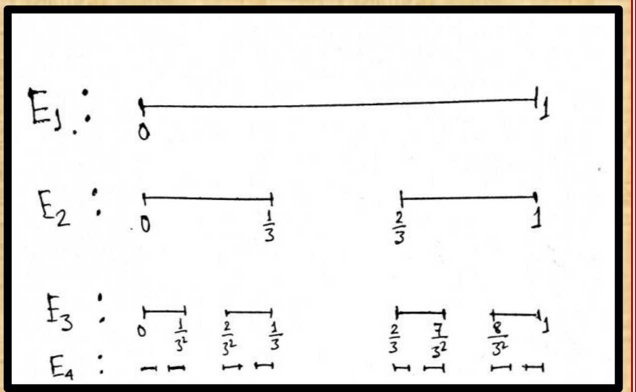
……

$E_n := \left[0, \frac{1}{3^n}\right] \cup \left[\frac{2}{3^n}, \frac{1}{3^{n-1}}\right] \cup \dots \dots \cup \left[\frac{3^n-1}{3^n}, 1\right]$.

……

Now, let, $C = \cap_{n=1}^\infty E_n$. Then, C is called the Cantor set.

A representation of the first four steps of the construction of the Cantor set is shown in the diagram:



2. TERNARY REPRESENTATION OF REALS AND EXPRESSION OF THE ELEMENTS OF THE CANTOR SET IN TERNARY FORM

We, in general, are accustomed to decimal representation of real numbers. If $x \in R$, we have the decimal representation of x, given by,

$$x = \cdots b_3 b_2 b_1 . a_1 a_2 a_3 \ldots = \cdots + 10^2 b_3 + 10 b_2 + b_1 + \frac{a_1}{10} + \frac{a_2}{10^2} + \cdots + \frac{a_n}{10^n} + \cdots (where,$$
$$a_i, b_i \in \{0,1,2, \ldots ,9\}, \forall\, i)$$

In the case of the computer language, we are accustomed with binary representation of real numbers, i.e.; we use the base 2 instead of 10. Thus, we get the expression for $x \in [0,1]$ as, $x = \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \cdots (where, a_i \in \{0,1\}, \forall\, i)$.

In each of the above two cases we basically divided each interval [0, 1] into 10 and 2 equal sub intervals respectively and denote them by 0,1,2, …, 9 in case of decimal and 0,1 in case of binary representation. Then for a number in the interval, we assigned a_1 as the interval number in which the number is located. Then we again divide this interval by 10 and 2 equal sub intervals respectively and assign a_2 as the interval number in which the number is located. Continuing this process we get the said expressions and the expression becomes unique for each number.

Now, what if we had divided the interval into 3 equal parts? Then we get the ternary representation of a number.

So, instead of taking 10 or 2 as base, if we take 3, then for $x \in [0,1]$, we get, $x = \sum_{i=1}^{\infty} \frac{a_i}{3^i}$, where, $a_i \in \{0,1,2\}, \forall\, i$. In fact, not only for elements in [0,1], but we can derive ternary representation for any real numbers and for two distinct real numbers the representations are also distinct (similar as decimal or binary representation).

Thus, we have the theorem: Every real number has a unique ternary representation.

Now, using this theory and by the construction of the Cantor set C as described earlier, we have the following obvious result:

Theorem: $x \in C$, if and only if, the ternary representation of x is given by, $x = \sum_{i=1}^{\infty} \frac{a_i}{3^i}, where\ a_i \in \{0,2\}$.

3. CARDINALITY OF CANTOR SET

The Cantor set, although is obtained from $[0, 1]$ by discarding uncountably many points, is still uncountable.

To prove this, let us assume that C is countable. Then, $C = \{a_n : n \in N\}$.

Then, $a_n = \sum_{i=1}^{\infty} \frac{x_{n_i}}{3^i}, where, x_{n_i} \in \{0,2\}, \forall\, n, \forall\, i$. [Using the ternary representation].

Now, construct, $:= \sum_{i=1}^{\infty} \frac{y_i}{3^i}, where,\ y_n = 2, if\ x_{nn} = 0\ and\ y_n = 0, if\ x_{nn} = 2$.

Then, $y \in C$, but $y \neq a_n, \forall\, n$, which contradicts that C is countable. Thus, C is uncountable (Proved).

4. AN UNCOUNTABLE SET WITH MEASURE ZERO

Measure is basically a generalization of geometrical measures (e.g.; length, area, volume etc.). We define outer measure of a set $E \subset R$ by, $m^*(E) = inf\ \{l(\Gamma) : \Gamma \in C(E)\}$, where, $C(E)$ is the set of all open covers of E, $l(\Gamma) = \sum_{I_n \in} \sum_{I_n \in \Gamma} l(I_n)$and $l(I_n)$ is the length of the interval I_n(Note that, $l(I) = b - a$, for any open or closed or half open half closed interval with boundary points a and b with $a < b$). Now, we say that E is measurable if $m^*(X) = m^*(X \cup E) +$

$m^*(X\setminus E), \forall\, E \subset R$, and then we denote the measure of E by $m(E)$ and then $m(E) = m^*(E)$.Now, we state the following results:

(i) $m^*(E) = 0 \Rightarrow E$ is measurable and $m(E) = 0$.

(ii) $m(I) = l(I)$, for any interval I.

(iii) For, a sequence of sets with $A_1 \supset A_2 \supset A_3 \supset \cdots$, $m^*(\cap A_n) = A_n$, where, $m^*(A_1) < \infty$.

(iv) For a disjoint collection of measurable sets, $m(\cup B_n) = \sum_n B_n$.

Now, we have Cantor set, $C = \cap_{n=1}^{\infty} E_n$. Also, clearly by construction of $E_n's$, using (ii), and (iv) $m^*(E_n) = \left(\frac{2}{3}\right)^n$.

So, by (iii), $m^*(C) = E_n$

$$= \lim_{n\to\infty} \left(\frac{2}{3}\right)^n = 0.$$

Hence, by (i), Cantor set is measurable and $m(C) = 0$ (Proved).

5. SOME OF THE TOPOLOGICAL PROPERTIES OF CANTOR SET

Here we discuss some topological property of the Cantor set, where the topology is the subspace topology on $C \subset R$, induced from usual topology on R.

- Cantor Set is Compact and T_2: Since, each E_n is finite union of closed intervals, so each $E_n$is closed, and hence, C being arbitrary intersection of closed set is closed. Moreover, $C \subset [0,1]$, and so bounded. Thus Cantor set is compact in real line (By, Heine-Borel theorem).

  Also, Cantor Set with usual topology is Hausdorff (T_2), being subspace of the T_2 space R. To prove this independently, directly from construction of the Cantor set, we have, for any two distinct points $x\ and\ y$ in C, there is $n \in N$ such that $|x - y| > \frac{1}{3^n}$. Then, $x\ and\ y$ are clearly in two different component of E_n, and thus can be separated by two disjoint open sets $in\ R, say, U\ and\ V$. Now, $x \in U\cap C\ and\ y \in V\cap C$, where $U\cap C$, $V\cap C$ are disjoint open sets in C, and thus C is T_2.

- $int(C) = \phi$:Since, $int(C) \neq \phi$ means there is an open interval $I \subset C$. But, $m(C) = 0 \Rightarrow m(I) = l(I) = 0$, which is not possible.

- Cantor Set is Perfect:We can prove that Cantor set is perfect set, i.e.; $C = C^d$, where C^d is the set containing all limits point of C.C being closed, $C^d \subset C$ is obvious. Conversely, for any $x = \sum_{i=1}^{\infty} \frac{x_i}{3^i}$, with $x_i = 0,2$, if we construct a sequence $\{y_n\}_n$ by $y_n = \sum_{i=1}^{\infty} \frac{y_{ni}}{3^i}$, with $y_{ni} = x_i$, $for\ i \neq n\ and\ y_{ni} = 2 - x_i, for\ i = n$,then one can easily check that this sequence converges to x, as $n \to \infty$.

- Any Compact Metric Space is Continuous image of Cantor Set: That is, for any compact metric space (X, d), there is a continuous surjection from $C\ to\ X$, where C is endowed with the usual Euclidean metric. A detailed proof of this is beyond the scope of this article, to study the detailed proof one can see Reference 1.

  There are also so many properties of Cantor Set, and each of them is more interesting than the other in some perspective. But, we shall end our discussion here by stating a Geometrical property of Cantor Set.

6. FRACTAL/SELF SIMILAR NATURE

A set is self similar if it has the property that at any magnification there are pieces of the sets that have the same shape as the whole set. From the figure drawn in page-1, and the construction of the

Cantor set, it can be said intuitively that, Cantor set is indeed a fractal or self similar set. For a detailed analytical proof one can see Reference 1.

7. CONCLUSION

Due to its mysterious behavior in so many perspectives, Cantor set has always been a topic of great interest among mathematicians. The concept of the Cantor Set has also extended to the α-Cantor set, Cantor-like set etc. The Cantor set, when extended to a two dimensional fractal figure using a square, instead of a line, is called Cantor dust. All of these sets also have their applications in many branches of Mathematics and they act as some rare counter examples also in many areas and one can expect that they will continue to amaze mathematicians in the future too by virtue of their properties.

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